The Twins' Paradox

Version 1
28th June, 2023

Introduction
Do not panic! I am not about to produce yet another argument showing how the Twins' Paradox proves Special Relativity wrong. I accept that the theory is correct, but believe that the two most common putative resolutions to the paradox are invalid. I will explain why I reject these explanations and I will then present my own resolution to the paradox. As usual I am not claiming to be the only person to have thought this way, only that these arguments are not discussed enough.

Overview of the paradox
I will start with a quick overview of the paradox. The first two things you learn about Special Relativity are that motion is relative and that time slows down when you are moving. This instantly leads to a paradox. If Alison is moving relative to Helena then Helena will measure Alison's clock as being slower. But from Alison's perspective Helena is moving relative to her. So she will measure Helena's clock running slower. So each will consider the other's clock to be slower.

I have seem some presentations where the mere slowing of time is the paradox. This is not true. If the two observers could agree on which one was slowing down then there would be no problem.

It is the contradiction in the two views that is the paradox.

The two views are shown below.

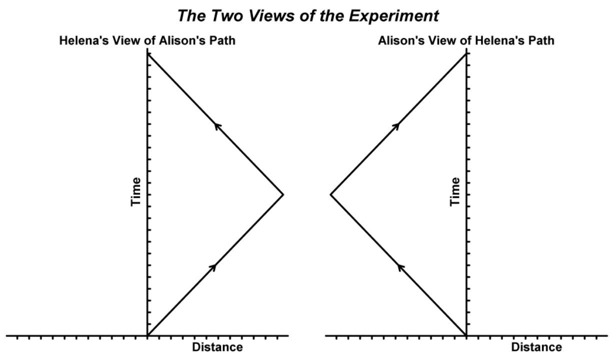

Figure 1

Note: in Special Relativity time is always the vertical axis.

The Asymmetry Argument
The most common argument used to defend relativity is to point out that the experiment is asymmetric. Acceleration is an absolute, if each twin were enclosed in a windowless box they could still use an accelerometer to deduce their own velocity profiles. Therefore the participants would know which one is really moving – hence whose clock correct.

Refuting the Asymmetry Argument
The asymmetry argument can be refuted simply by making the journeys symmetric. We will leave

Helena at home to act as a reference and bring in another twin, Beth. Beth will travel in the opposite direction to Alison and follow the same velocity profile – except with the direction reversed.

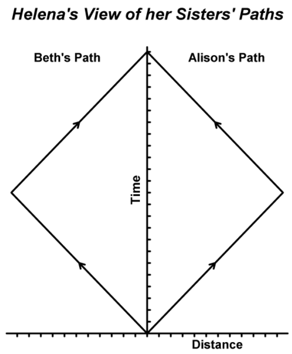

Figure 2

Now at any point in time (however the various travellers measure time) each sisters will see the other as being in one of three states: moving away from her; at rest relative to her (because they are moving in the same direction at the same time); or moving towards her. In the two cases of relative motion the direction of motion will be irrelevant and so each traveller will measure the clock of the other as running slow. During the phases of identical motion the other traveller's clock will appear to run at the same speed. So overall the total effect will be for the two travellers to measure the clock of the other as running slow even in this symmetric experiment – so the paradox persists.

The Acceleration Argument

The second counterargument invokes the effects of the acceleration phases. This usually involves the time related effects of General Relativity. The idea is that the acceleration effects cancel out the velocity effects leading to no overall difference in the total elapsed time.

Refuting the Acceleration Argument

The way to counter this argument is to do the experiment twice. In the second run of the experiment the travellers will follow the same acceleration profiles with the only difference being the duration of the constant velocity part. In the second case the travellers will maintain the constant velocity for twice as long.

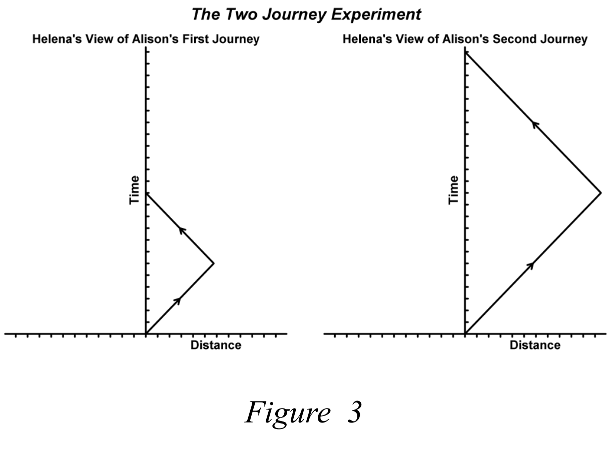

Figure 3

This will lead to twice the dilation effect in the constant velocity phases. But as the acceleration profiles in both runs are the same, the proposed cancelling effect will be the same – so it cannot cancel two different dilation effects. It should also be noted that acceleration also causes a slowing down of time, so could not cancel out another time dilation effect.

A more general point is that we are considering the internal consistency of a specific theory, so you cannot add other theories to resolve it. A theory can be incorrect yet still be paradox free (such as a Newtonian analysis of the twins experiment). When performing the experiment for real there will be a contribution from General Relativity but this will not be relevant to the paradox.

Another approach is to have four travellers, with all travelling at a constant velocity throughout the experiment – although starting at different places and travelling in different directions. The originals, Alison and Beth travel towards Helena at the same constants speed but from opposite directions, such that they reach Helena at the same time. At this point they synchronise their clocks and the experiment starts. While this is happening Rachel is travelling towards Alison along Alison's path and Sarah is travelling towards Beth along Bath's path.

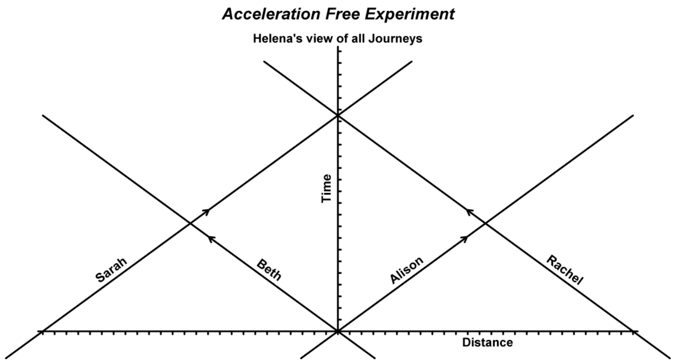

Figure 4

When Alison and Rachel cross Rachel synchronises her clock with Alison's. Similarly when Sarah crosses with Beth she synchronises her clock with Beth. This way no traveller accelerates during the experiment. Thus Rachel is actually performing Alison's return journey without any deceleration or acceleration occurring – and the same for Sarah and Beth.

Alternative Resolution

The explanation I came up with involves comparing the system with what I will 'Rotational Relativity'. This is a simple two dimensional geometric transform. The transformation involves rotating a frame of reference.

We will take two frames rotated relative to each other.

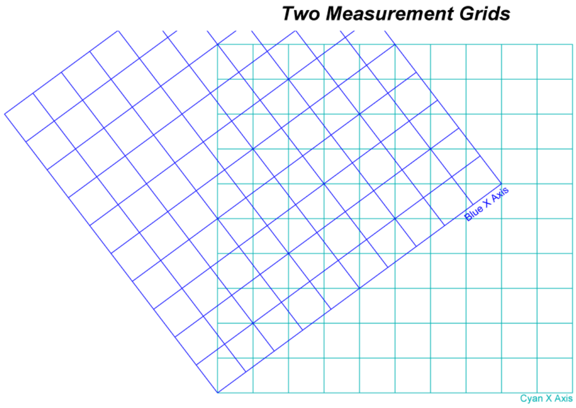

Figure 5

Let us place a rod along the X axis of the blue frame :-

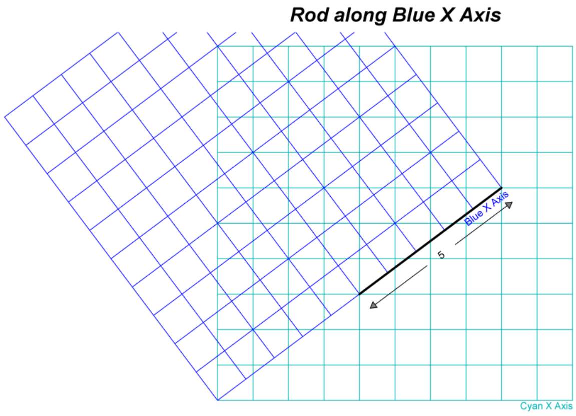

Figure 6

If we consider the X-length to be the difference in the X coordinates of the two ends then we get a length of 5 units in the blue frame. If we now consider how the cyan frame see things :-

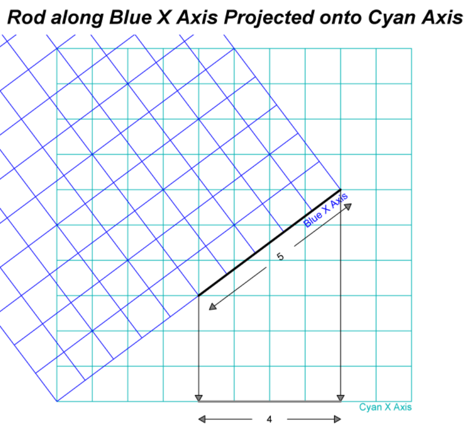

Figure 7

We will see that the projected length is shorter (in this case with and angle of about 38 degrees - the angle whose cosine is 0.8 - the projected length is 4 units). So a rotated rod appears shorter. But rotation is relative, so a rod in the cyan frame should appear shorted in the blue frame :-

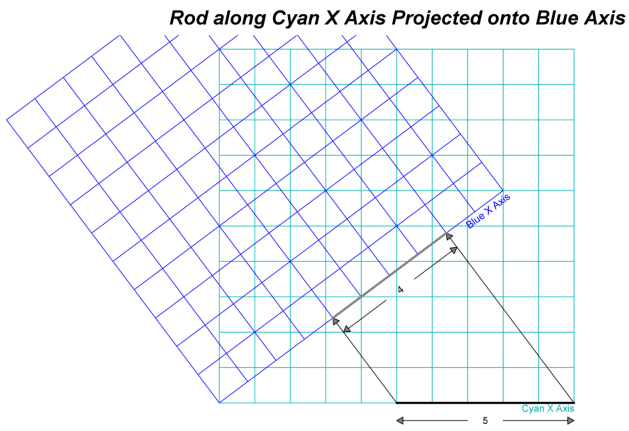

Figure 8

So each sees objects in the other frame to be contracted. But it is easy to see why there is not a problem. The same applies in Special Relativity.

Resolving the Twins' Paradox
This was the clue that lead to my resolution of the twins' paradox.

I shall now present a resolution to a an equivalent of the twins' paradox in the Rotational Relativity system.

So far we have only labelled the horizontal axis. We will now label the vertical axis as T and call the units along that axis 'ticks'. Remember in Special Relativity the time axis is always vertical. We can consider a line on the plane as representing the path of a traveller over time, but we will stick with a purely geometric analysis. We will measure the displacement along the various T axes as measured in ticks.

We will start by considering a geometric equivalent of Alison's complete round trip as seem from Helena's frame.

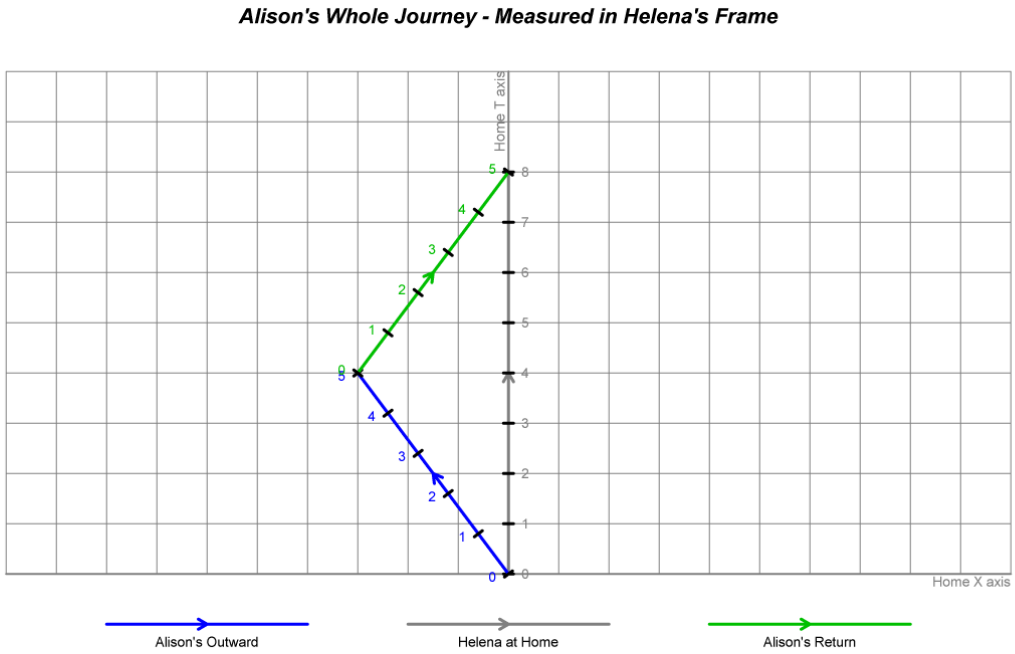

Figure 9

There you will see that the total number of ticks along Alison's sloping path is 10 (5 + 5). But Helena's path has only 8 ticks. The intervals in Alison's frame, when projected onto Helena's frame, are shorter, so the ticks are shorter in the rotated frame (note: this is the opposite of Special Relativity where units of time are longer). But a rotation is relative so Alison's could consider herself to be at 'rest' – i.e. oriented with her T-axis vertical rather than at an angle. So let us analyse that situation. As Alison actually travels in two different frames we will need to consider each in turn, then add the results.

View from Alison's Outward Journey

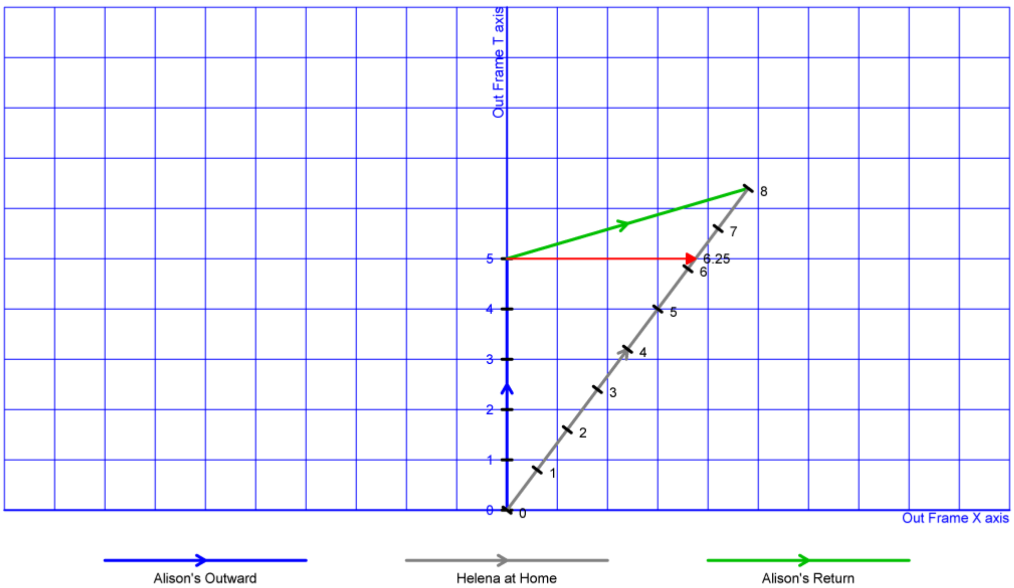

Figure 10

In the outward phase of Alison's journey her path covers 5 ticks, but she measures Helena's path covering just over 6 ticks. So Helena's 'clock' is ticking faster by a factor of 1.25. Let us now look at Alison's return journey.

View from Alison's Return Journey

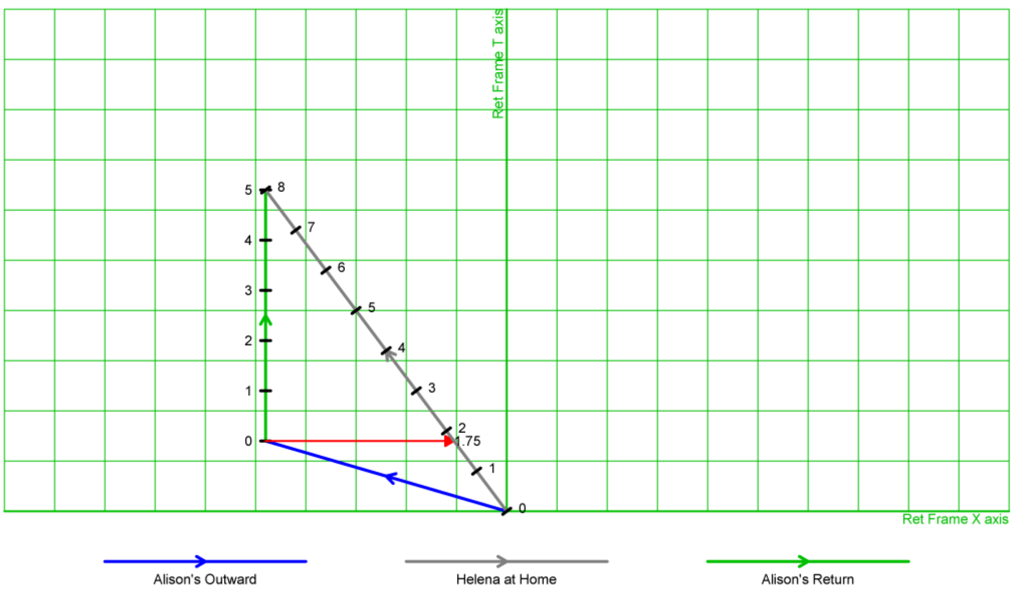

Figure 11

During the return journey Alison also measures Helens 'clock' ticking faster and measures a total of 5 ticks for herself and 6.25 ticks (1.75 to 8.00) for Helena. So for the whole journey Helena measures Alison's elapsed 'time' as greater and Alison also measures Helena's elapsed 'time' as longer.

But you should notice that during both phases of the journey Alison's measurement includes the section from 1.75 to 6.25 in Helena's world line. Thus she is double counting a total of 4.5 ticks. If you deduct these from Alison's total measurement she is left with 8 ticks in Helena's world line – which is what Helena herself measures. So the reason for the apparent paradox is the double counting.

A number of books do mention this but they try to save space by plotting everything on a single diagram, thus making the explanation harder to understand.